# How Risk Averse and how Prudent are Workers?*

Robin Jessen†
Davud Rostam-Afschar‡

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**Abstract**

Analogous to the widely studied concept of precautionary savings, our study contributes to the burgeoning literature on precautionary labor supply behavior when wages are subject to uninsurable wage risk. We estimate the parameters of a dynamic structural model of labor supply of men. We test whether workers are risk averse and prudent in the sense that they increase labor supply in anticipation of higher wage risk. The specification provides a direct test of the widely used CRRA utility function and could identify the Frisch elasticity. Using data from the Panel Study of Income Dynamics (PSID) and the German Socio-Economic Panel (SOEP), we are able to replicate the findings of the previous literature. However, with an extended specification that includes variability of leisure and wage, we reject restrictions implied by CRRA utility for both countries.

**Keywords** Labor Supply · Wage Risk · Frisch Elasticity · Prudence · CRRA utility

**JEL Classification** D91 · J22

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†Department of Economics, Freie Universität Berlin, Boltzmannstr. 20, 14195 Berlin, Germany (e-mail: r.jessen@fu-berlin.de).

‡Department of Economics, Freie Universität Berlin (e-mail: davud.rostam-afschar@fu-berlin.de).

# 1 Introduction

In a life cycle model where households face uninsurable uncertainty in wages, precautionary saving is one way to insure against shocks. Low (2005) stresses that another important channel is to adjust hours of work out of precaution. This implies that households do not only cut consumption to build a stock of buffer savings but also cut leisure. While precautionary saving is widely studied, precautionary labor supply is relatively unexplored. As reported by Mulligan (1998, p. 1034), “there is no empirical evidence that precautionary motives for delaying leisure are important”. However, Pistaferri (2003) and Parker et al. (2005) find some evidence that uncertainty is related to hours worked. To study this effect, we build on MaCurdy (1981) and Altonji (1986), and reproduce the results in Domeij and Flodén (2006), a more recent study using the same specification.[1] Then, we extend the model allowing for an effect of wage risk to test whether this affects the conventional estimates of the Frisch elasticity. Our specification provides a direct test of the implications of preferences of the constant relative risk aversion (CRRA) form. We compare our results from the Panel Study of Income Dynamics (PSID) to those obtained with the German Socio-Economic Panel (SOEP) using the same specifications and can therefore compare the preference parameters estimated for the US to those in Germany where wage risk may be mitigated in a different way though the tax system.

We use an analogy from precautionary savings and estimate the structural parameters measuring risk aversion and prudence defined (for consumption in Kimball (1990) and) for leisure in Flodén (2006). Similarly to precautionary saving, the conditions that households defer leisure due to uncertainty are that the second derivative of instantaneous utility with respect to labor is negative, i.e. workers are risk averse, and that the third derivative with respect to leisure is positive, i.e. workers are prudent. This means that not only the marginal utility of leisure is higher when leisure is low, but also that the rate at which the marginal valuation rises when leisure falls is greater when leisure is low than when it is high. This implies that individuals react to increases in uncertainty by increasing labor supply. Thus, both precautionary saving and labor supply are devices to smooth combinations of consumption and leisure. The direct policy implication of our study is that policies that aim at reducing uncertainty, such as minimum wages, progressive taxation or transfers influence labor supply not only by changing incentives in well studied ways, but also by reducing the precautionary labor supply motives.

[1] See Keane (2011) and Blundell and Macurdy (1999) for reviews of approaches to estimating the Frisch elasticity.

If precautionary labor supply is important, then part of wealth that studies like Carroll and Samwick (1997, 1998) attribute to the extra uncertainty might be due to cuts in leisure, not consumption. However, since empirical evidence on precautionary saving is mixed (Guiso et al. (1992); Dynan (1993); Lusardi (1998); Fossen and Rostam-Afschar (2013)), more research needs to investigate the relative importance of precautionary savings and precautionary labor supply.

Unfortunately there is no analytical solution for a multi-period model where households can choose both consumption and labor supply. This problem is also present in modern models of consumption choice where labor supply is exogenous. A frequently used method to study precautionary saving is therefore to approximate the Euler equation and empirically estimate some of the structural parameters (see Dynan (1993); Browning and Lusardi (1996); Attanasio and Low (2004); Gruber (2013)).[2] We apply this approach to the Euler equation which describes how leisure grows in the life cycle because this allows us to directly estimate and test the structural parameters.

In a first step, we reproduce the results in Domeij and Flodén (2006) and show that the estimated Frisch elasticities are very sensitive to the choice of the instruments using PSID data. Then we repeat the analysis with German data from the SOEP to study differences in labor supply behavior across these countries. With our preferred specification we obtain statistically significant positive Frisch elasticities that are comparable to those obtained in the previous literature.

Using the extended estimation specification, we reject the implied restrictions of CRRA utility both for the US and for Germany. This suggests that the widely used assumption of CRRA utility is a poor approximation of economic behavior.

The next section derives the empirical specification from theoretical considerations, Section 3 briefly describes the data, and construction of key variables. Section 4 presents estimation results, and Section 5 concludes.

# 2 Theoretical Considerations and Empirical Strategy

## 2.1 Approximation of Optimal Behavior

Individuals maximize expected utility by choosing consumption and leisure for each period $t$ of life which ends in $T$. Utility is additively separable across time periods and within periods. $\beta$ is a

[2]One approach is to *assume* a utility function and use numerical techniques instead of an approximation of the Euler equation (Carroll (2001)) to simulate policy experiments (Low 2005). However, this does not allow to *test* the implications of the utility function.

discount factor.

$$\max_{c_t,l_t} E_{t_0}\left[\sum_{t=t_0}^{T}\beta^{t-t_0}u(C_t,L_t)\right],$$

subject to

$$A_{t+1}=(1+r)[A_t+(\bar{L}-L_t)W_t-C_t].$$

Real wages $W_t$ are uncertain and uninsurable, the real interest rate $r$ is constant. $C_t$ denotes consumption, $A_t$ the amount of assets held at the start of period $t$ and $\bar{L}$ total time endowment, which is spent on leisure $L_t$ or work.

Assuming an interior solution for leisure, the first order conditions of the maximization problem are the Euler function for consumption, where $\lambda_t$ denotes marginal utility of wealth, and the condition that the marginal rate of substitution equal the marginal rate of transformation:

$$\frac{\partial u(C_t,L_t)}{\partial C_t}=\frac{\lambda_t}{\beta^{t-t_0}}=\beta(1+r)E_{t_0}\left[\frac{\partial u(C_{t+1},L_{t+})}{\partial C_{t+1}}\right], \tag{1}$$

$$\frac{\partial u(C_t,L_t)}{\partial L_t}=W_t\frac{\partial u(C_t,L_t)}{\partial C_t}.$$

We substitute to obtain

$$\frac{\partial u(C_t,L_t)}{\partial L_t}\frac{1}{W_t}=\beta(1+r)E_{t_0}\left[\frac{\partial u(C_{t+1},L_{t+1})}{\partial L_{t+1}}\frac{1}{W_{t+1}}\right]. \tag{2}$$

Assuming additive separable constant relative risk aversion (CRRA) utility with

$$u(C,L)=\frac{C^{1-\gamma}}{1-\gamma}+B\frac{L^{1-\rho}}{1-\rho}, \tag{3}$$

the parameter of relative risk aversion with respect to leisure $\rho$ (Pratt 1964) is defined as $-L\frac{u_{LL}}{u_L}=\rho$ and the parameter of relative prudence with respect to leisure $\rho+1$ is defined as $-L\frac{u_{LLL}}{u_{LL}}=\rho+1$ (Kimball (1990); Flodén (2006)).

Since $u_L=L^{-\rho}$ and under certainty, taking logs of equation (2) gives the estimation equation in MaCurdy (1981)

$$\Delta\ln L_{t+1}=\frac{1}{\rho}[(r-\delta)-\Delta\ln W_{t+1}]. \tag{4}$$

Rewrite equation (2) again for CRRA preferences with $u_L=L^{-\rho}$ but this time under uncertainty and define as $\lambda_{t+1}$ the future marginal utility of wealth with expectation error $e_{\lambda t+1}$ such that $\lambda_{t+1}+e_{\lambda t+1}=\beta^{t+1-t_0}E_{t_0}\left[\frac{\partial u(C_{t+1},L_{t+1})}{\partial L_{t+1}}\frac{1}{W_{t+1}}\right]$. Then, taking logs and applying a first order Taylor approximation to $\ln(\lambda_t-e_{\lambda t})$ around $e_{\lambda t}=0$ gives $\ln\lambda_t-\frac{e_{\lambda t}}{\lambda_t}$. Defining $u_t$ as $-\frac{e_{\lambda t}}{\lambda_t}$ yields the estimation equation in Altonji (1986) (see Peterman (2014) for a replication):

$$\Delta \ln L_{t+1} = \frac{1}{\rho}[(r-\delta) - \Delta \ln W_{t+1} + u_t]. \tag{5}$$

Pistaferri (2003) uses this approximation and assumes a specific wage process[3] to estimate the effect of subjective income risk on labor supply.

Using the same procedure as in Altonji (1986), but applying a *second* order approximation instead, gives

$$\Delta \ln L_{t+1} = \frac{1}{\rho}\left\{(r-\delta) - \Delta \ln W_{t+1} - \frac{e_{\lambda t}}{\lambda_t} - \frac{1}{2}\left[\frac{e_{\lambda t}}{\lambda_t}\right]^2\right\}. \tag{6}$$

This equation is analogous to Browning and Lusardi (1996); Gruber (2013)[4] for consumption and equivalent to equation (15') in Domeij and Flodén (2006) if borrowing constraints are assumed not to produce a bias. Note that the Frisch elasticity is the coefficient on both wage growth and one half of the square of the forecast error divided by the marginal utility of wealth.

These specifications do not allow to estimate the coefficient of relative risk aversion and relative prudence in one specification. Moreover, $\frac{e_{\lambda t}}{\lambda_t}$ and $\frac{1}{2}\left[\frac{e_{\lambda t}}{\lambda_t}\right]^2$ are not observed. Not including these variables in the estimation equation leads to an omitted variable bias if at least one of these variables is correlated with wage growth.

Blundell et al. (2012) use a different approach. The first difference is that they apply a second order Taylor approximation to a different function, namely $\exp \ln \lambda_{t+1}$ around $\ln \lambda + \delta - r$ instead of $\ln(\lambda_t - e_{\lambda t})$ around $e_{\lambda t} = 0$. This gives the following expression for the expected growth of the marginal utility of wealth:

$$E_{t_0} \Delta \ln \lambda_{t+1} \approx (\delta - r) - \frac{1}{2} E_{t_0}(\Delta \lambda_{t+1} - (\delta - r))^2.$$

In a second step, that is different form the other studies, a first order Taylor approximation is applied to the marginal utility of leisure $u_L(L_{t+1})$ around $L_t$. This gives

$$\Delta \ln u_L(L_{t+1}) \approx -\frac{u_{LL}(L_t)L_t}{u_L(L_t)} \Delta \ln L_{t+1}.$$

Therefore, with $u_L(L_{t+1}) = \lambda_{t+1} W_{t+1}$,

$$\Delta \ln L_{t+1} \approx -\frac{u_L(L_{t+1})}{u_{LL}(L_{t+1})L_{t+1}}(\Delta \ln \lambda_{t+1} + \Delta \ln W_{t+1}),$$

[3] See Appendix A for a similar process.

[4] An alternative is to assume that $e_{\lambda t}$ is log-normally distributed in which case the approximation is exact.

where $\Delta \ln \lambda_{t+1} \approx (\delta - r) - \frac{1}{2}\Delta\lambda_{t+1} - (\delta - r)^2 + \varepsilon_{t+1}$.

This specification is not estimated since $\lambda_{t+1}$ is unknown. Therefore, the intertemporal budget constraint is approximated in order to estimate the parameters of the model.

Our specification differs from the aforementioned ones, since we obtain it by approximating the first order condition using the second order Taylor expansion of $\frac{u(C_{t+1},L_{t+1})}{\partial L_{t+1}}\frac{1}{W_{t+1}}$ around $C_t$, $L_t$ and $W_t$. This gives a different specification (see Low (2005)) similar to e.g. Ludvigson and Paxson (2001); Carroll (2001) who approximate around the point $C_t$.[5]

$$\frac{\partial u(C_{t+1},L_{t+1})}{\partial L_{t+1}}\frac{1}{W_{t+1}} \approx u_L\frac{1}{W_t} + u_{LL}\frac{1}{W_t}(\Delta L_{t+1}) - u_L\frac{1}{W_t}(\Delta \ln w_{t+1}) \tag{7}$$
$$+\frac{1}{2}u_{LLL}\frac{1}{W_t}(\Delta L_{t+1})^2 + \frac{1}{2}u_L\frac{1}{W_t}(\Delta \ln w_{t+1})^2.$$

Multiplying the expression with $W_t/u_{LL}$, taking expectations, and substituting back into (2)[6] and defining $\delta = 1/\beta - 1$ gives

$$E_{t_0}\Delta L_{t+1} \approx -\frac{u_L}{u_{LL}}(r-\delta) - \left(-\frac{u_L}{u_{LL}}\right)E_{t_0}\Delta \ln W_{t+1} \tag{8}$$

$$+\frac{1}{2}\left(-\frac{u_{LLL}}{u_{LL}}\right)E_{t_0}[(\Delta L_{t+1})^2] + \frac{1}{2}\left(-\frac{u_L}{u_{LL}}\right)E_{t_0}[(\Delta \ln W_{t+1})^2].$$

The first two terms capture the effect of $r > \delta$ and of the expectation of the change in wages. If $r > \delta$ leisure decreases with time; if wages are expected to increase, leisure is expected to decrease. The effect of uncertainty is captured by the last two terms: The direct precautionary effect that increased variability in leisure leads to leisure being deferred and an indirect effect that variability in wage leads to leisure being deferred. In summary, the approximations suggest, that precautionary motives lead individuals to work more today, and to work less in the future in the presence of wage uncertainty and variability in leisure.

## 2.2 Empirical Specification

Backdating equation (8), using CRRA utility, dividing by $L_t$ on both sides, assuming linear (conditional) expectations, we obtain our estimation equation. This specification may be augmented by

[5] Note that cross derivatives are zero due to additive separability.

[6] Note that for small values of $\delta$ and $r$, $1 - \frac{1}{\beta(1+r)} = 1 - \frac{1+\delta}{1+r} \approx r - \delta$ with $\beta = \frac{1}{1+\delta}$ and that $\frac{w_{t+1}-w_t}{w_t} \approx \Delta \ln w_{t+1}$.

including taste-shifting individual characteristics $\psi x'_{it}$ (age, its square, profession, marital status, children, virtual income) and an unobserved effect $\alpha_i$,

$$\begin{aligned}\Delta \ln L_{it} &\approx \beta_0 + \beta_1 \Delta \ln W_{it} + \beta_2 \frac{1}{2} (\Delta \ln W_t)^2 \\ &+ \beta_3 \frac{1}{2L_{it-1}^2} (\Delta L_{it})^2 \\ &+ \psi \mathbf{x}'_{it} + \alpha_i + \varepsilon_{it}.\end{aligned} \tag{9}$$

The estimation equations represent relationships of endogenous variables that must hold if the model is correct. They are not directly estimable as the term containing leisure is a choice term. Moreover, wage uncertainty could depend on leisure choice. Hourly wages are constructed by dividing annual labor income by hours. If hours are measured with error, this introduces a negative correlation between wage and hours (denominator bias, see Altonji 1986). Moreover, the expectation error is correlated with wage changes if the latter are not known in advance. [7] For the estimation, it is necessary to instrument all endogenous terms with variables that are uncorrelated with taste shocks $\varepsilon_{it}$. We use lags of the changes of variability in wages and leisure as instruments for variability terms.

MaCurdy's suggestion to use human capital related instruments (family background variables, education, age, interactions between education and age, and dummy variables for each year of the sample, IV Human Capital) has sparked a debate about the choice of instruments. These are likely uncorrelated with measurement error as well as changes in $ln\lambda_t$ as they are known in advance. However, Altonji (1986) pointed out that the instruments have a weak role in explaining wage changes. Additionally, age is likely to be correlated with changes in taste, which enter the error term, and is therefore an invalid instrument. Additionally, Altonji (1986) points out that the amount of time on the job invested in human capital decreases while the wage increases with age. Therefore, the effect of age on wage growth will be negatively correlated with its effect on the change in future earnings potential from an additional hour of work at the job today leading to a downward bias in elasticity estimates. These problems are addressed by using the lagged change in *directly asked* hourly wage as instrument for the change in wage.

Unfortunately, hourly wages are not asked directly in the SOEP. Therefore, we use the lagged constructed hourly wage as instrument. If the measurement error in hours is uncorrelated across

[7] Note that extrapolating current net wages using changes in the tax system and using these extrapolations as instruments is not valid if the changes in the tax system are not fully anticipated. Un-anticipated changes have an impact on the change of $\lambda$.

periods, this instrument is not prone to the denominator bias. As an alternative instrument, we use lagged labor income which contains information that individuals use to form expectations about wage growth, while the denominator bias is not present.

Most of the above specifications abstract from progressive taxation and transfers. Only e.g. Ziliak and Kniesner (1999) and Blundell et al. (2012) account for taxes, the former calculate "virtual wealth" while the latter use a function with two parameters used in the public economic and macroeconomic literature frequently (see e.g. Feldstein 1969; Bénabou 2002; Heathcote et al. 2014). For the estimation with German data, we calculate marginal net wages with the tax and transfer calculator (STSM) Jessen et al. (2015).

# 3 Data

We use data from 1983 to 1995 from the Panel Study of Income and Dynamics (PSID), a representative survey of the US population, to estimate the model. We restrict the sample in the same way as Domeij and Flodén (2006) in order to reproduce their results. This includes dropping households with missing observations and those with unrealistic values for hours or wages or very large increases or decreases, "jumps" likely due to measurement errors. See Domeij and Flodén (2006) for more information. The estimation sample consists of males between 25 and 61 years of age.

We use the 100 percent research sample of German Socio-Economic Panel (SOEP) 1996-2011, also known as GSOEP. Wagner et al. (2007) provide a detailed description of the data. We restrict our sample in the same way as the PSID to obtain a comparable dataset. This is possible since these datasets are linked through the Cross-National Equivalent File (CNEF). We use the most recent waves from the SOEP covering the years 1996 to 2011. For these waves, we can use a tax transfer simulation model (STSM) to calculate net marginal wages. We restrict the analysis with the PSID to gross wages.

Table 1: Summary Statistics PSID and SOEP.

| | *Mean* | | *SD* | | *Min* | | *Max* | |
|---|---|---|---|---|---|---|---|---|
| | PSID | SOEP | PSID | SOEP | PSID | SOEP | PSID | SOEP |
| $\frac{inc}{h}$ | 10.65 | 11.49 | 4.72 | 4.71 | 0.63 | 2.44 | 47.76 | 32.72 |
| $W$ | 9.68 | | 4.83 | | 2.89 | | 68.84 | |
| $\Delta\ln(inc/h)$ | 0.017 | 0.013 | 0.231 | 0.222 | -0.914 | -0.913 | 1.248 | 1.246 |
| $\Delta\ln W$ | 0.001 | | 0.126 | | -0.843 | | 1.082 | |
| $\frac{1}{2}\Delta\ln(inc/h)^2$ | 0.027 | 0.025 | 0.067 | 0.057 | 0 | 0 | 0.779 | 0.777 |

*Continued on next page*

| | *Mean* | | *SD* | | *Min* | | *Max* | |
|---|---|---|---|---|---|---|---|---|
| | PSID | SOEP | PSID | SOEP | PSID | SOEP | PSID | SOEP |
| $\frac{1}{2}(\Delta \ln W)^2$ | 0.008 | | 0.033 | | 0 | | 0.585 | |
| $(\frac{inc}{h})^{\text{Net}}$ | | 6.49 | | 2.85 | | 0.17 | | 19.24 |
| $\Delta \ln[(inc/h)^{\text{Net}}]$ | | 0.016 | | 0.263 | | -0.915 | | 1.246 |
| $\frac{1}{2}\Delta \ln[(inc/h)^{\text{Net}}]^2$ | | 0.035 | | 0.072 | | 0 | | 0.776 |
| $\frac{1}{2L^2}(\Delta L)^2$ | 0.017 | 0.009 | 0.092 | 0.074 | 0 | 0 | 1.805 | 4.007 |
| *inc* | 22,957 | 25,100 | 10,212 | 11,230 | 867 | 1,373 | 80,469 | 116,894 |
| *leisure* | 2,817 | 2,805 | 442 | 374 | 500 | 662 | 4,480 | 4,791 |
| *age* | 39 | 43 | 9 | 9 | 25 | 27 | 61 | 60 |
| *educ* | 12 | 12 | 2 | 3 | 3 | 7 | 17 | 18 |
| *N* | 1,277 | 44,668 | | | | | | |

*Notes:* $\frac{inc}{h}$ and *inc* are measured in 1984 Dollars and in 1984 Euro, respectively, $W$ in 1984 Dollars. *leisure* is measured in annual hours, *age* and *educ* in years.

*Source:* Authors' calculations based on the PSID (1983-1995) and the SOEP (1996-2011).

Table 1 shows summary statistics of the variables that we include in our specifications. The first variable is the constructed hourly wage, $\frac{inc}{h}$, i.e. individual annual labor income divided by individual annual hours worked. $W$ denotes directly asked hourly wage (only in the PSID), $\Delta \ln(inc/h)$ and $\Delta \ln W$ represent the growth rates of these wage measures. $\frac{1}{2}\Delta \ln(inc/h)^2$ and $\frac{1}{2}(\Delta \ln W)^2$ show the square of the respective growth rate times 0.5. $^{\text{Net}}$ indicates net marginal wages. The term $\frac{1}{2L^2}(\Delta L)^2$ measures the squared change of leisure divided by one half of squared leisure, a variable derived in Section 2.1. Individual annual pre-government labor income is denoted by *inc*, individual annual work hours by *leisure*, age by *age*, and years of education by *educ*.

Before turning to the empirical analysis, it is instructive to describe the growth pattern of hours and wages over the life cycle. Figures 1 and 2 show the age profile of annual hours of work and hourly wages for 9 cohorts whose membership is defined according to the year of birth. The profiles are constructed from repeated cross-sections of the PSID from 1970 to 2009 and from the SOEP from 1984 to 2012 where only married males aged 25 to 60 populate the sample. The hours profile was slightly increasing for the US and slightly declining for Germany over the life cycle. The age profile of the hourly wage rate is hump-shaped in both countries, and the wage decline occurs relatively late in the life cycle (at around age 50). Note that in the raw data, hours and wages are negatively correlated.

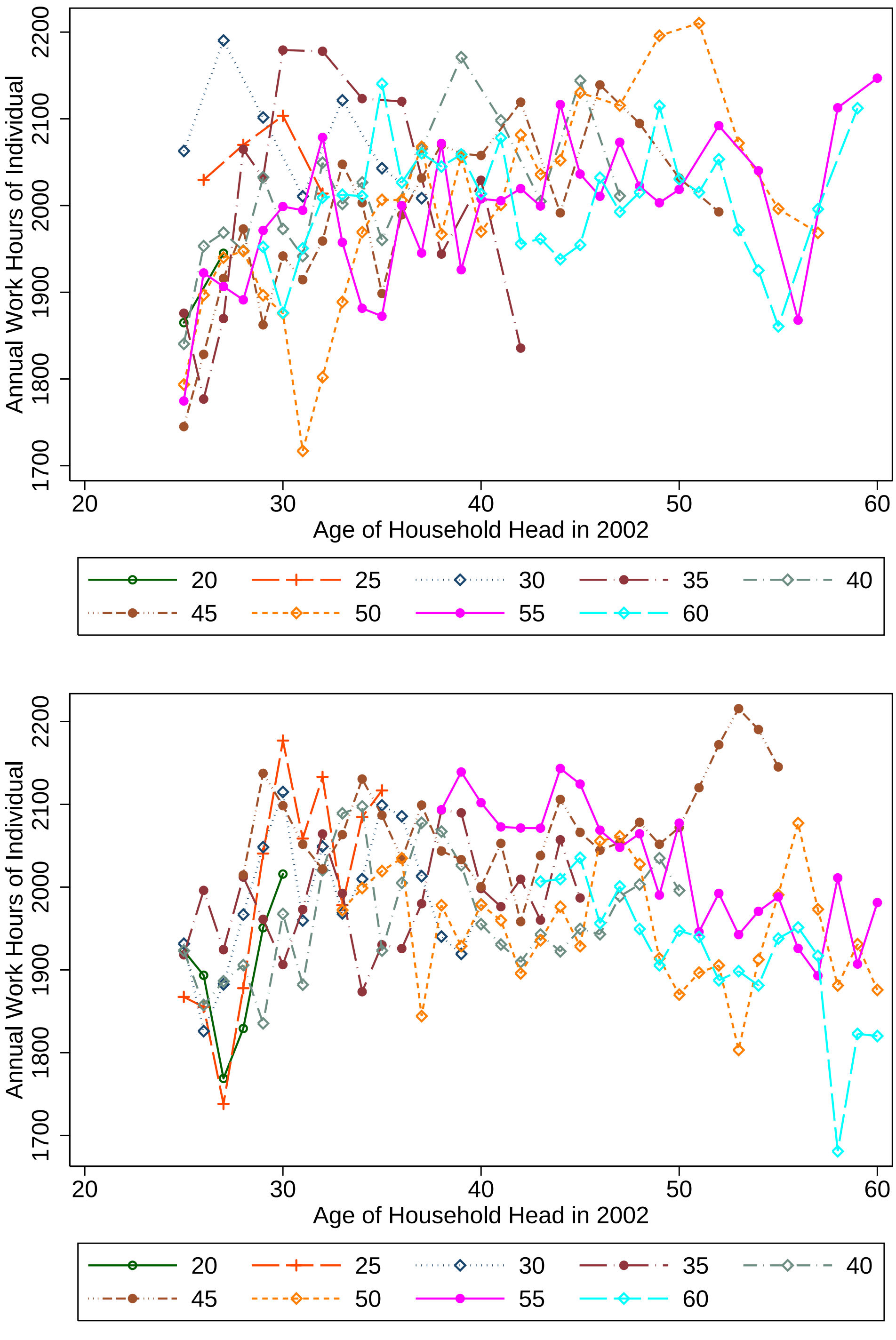


Figure 1: Hours over Age for 9 Cohorts.

*Source:* Own calculations based on the PSID (1970-2009) (upper panel) and the SOEP v29l (1984-2012) (lower panel).

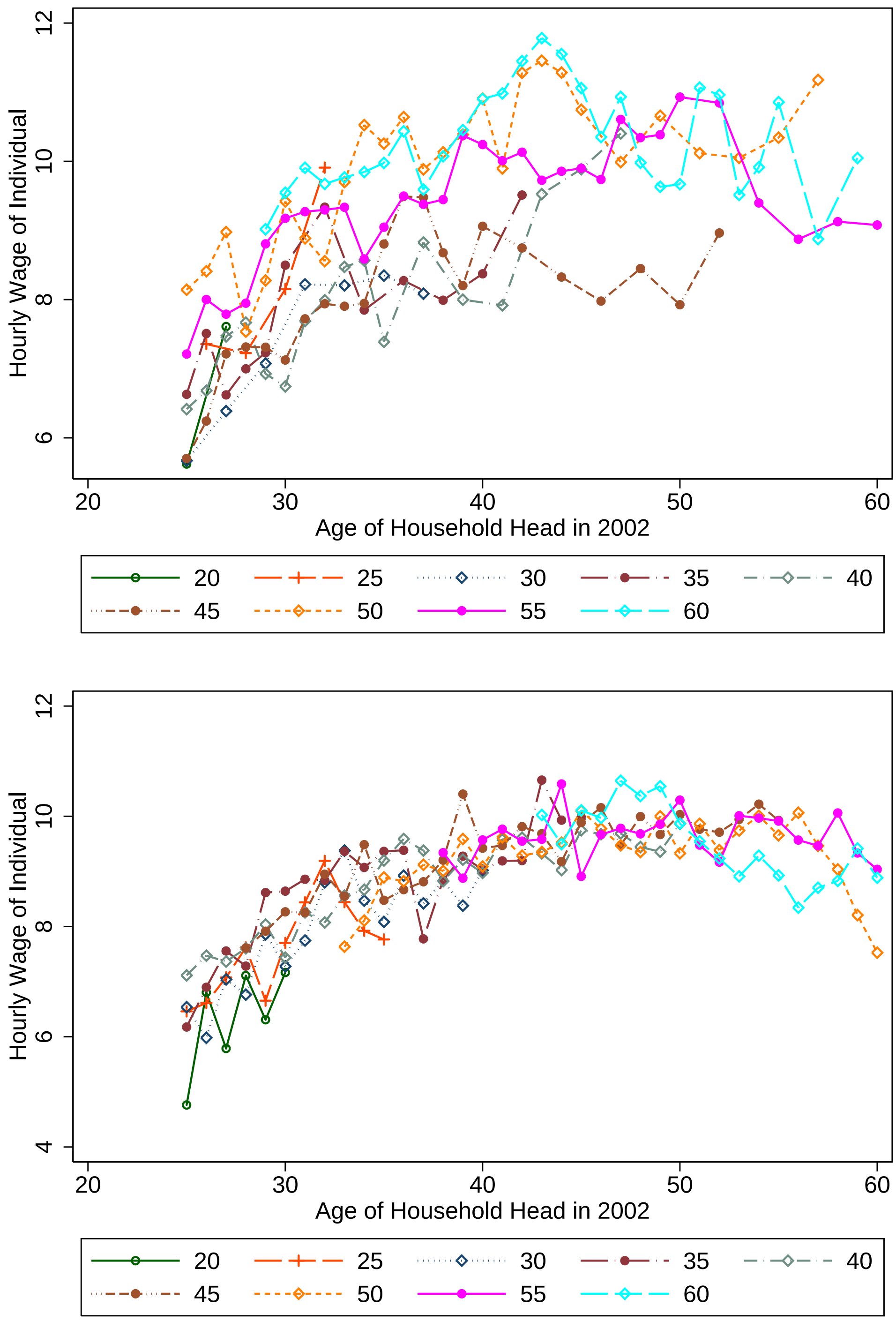


Figure 2: Wage over Age for 9 Cohorts.

*Source:* Own calculations based on the PSID (1970-2009) (upper panel) and the SOEP v29l (1984-2012) (lower panel).

# 4 Estimation Results

First, we replicate the results in Domeij and Flodén (2006) using the estimation equation (5) and show how the choice of instruments affects the results using the original PSID dataset. In column (1) of Table 2, we follow Altonji (1986); Domeij and Flodén (2006) and use the lagged level and the lagged difference of the log of the directly asked hourly wage in a specification where the dependent variable is the growth rate of *hours* as used in most of the literature. With this specification, we obtain exactly the same result as Domeij and Flodén (2006).

Table 2: Replication and Alternative Regression of Growth Rate of *Hours* Using PSID Data.

| | Domeij and Flodén | IV $\frac{inc_{t-1}}{h_{t-1}}$ | IV $inc_{t-1}$ | IV HC |
|---|---|---|---|---|
| | (1) | (2) | (3) | (4) |
| $\Delta \ln(inc/h)$ | 0.159 | -0.186*** | 0.483*** | 0.478 |
| | (0.132) | (0.050) | (0.116) | (0.295) |
| Constant | 0.008 | 0.014** | 0.002 | 0.021** |
| | (0.006) | (0.005) | (0.007) | (0.007) |
| *N* | 1,277 | 1,277 | 1,277 | 2,865 |

*Notes:* Standard errors in parentheses. Column (1) reproduces the results in the upper panel (Full Sample) of Table 5 in Domeij and Flodén (2006). Instruments are the lagged level and the lagged difference of the log of the directly asked hourly wage. Dependent variable is the growth rate of hours. In column (2), instruments are the lagged level and the lagged difference of the log of the constructed hourly wage. In column (3), instruments are lagged levels and lagged differences of the log of annual labor income. In column (4), instruments are age, age squared, education, education squared, interactions between age and education, and age and education squared.

*Source:* Authors' calculations based on the PSID (1983-1985, 1988-1990, 1993-1995).

In the second column, we use the lagged level and the lagged difference of the log of the constructed hourly wage. In contrast to the first column, this instrument yields a statistically significant coefficient of wage growth. However, the sign is negative, which suggests that the denominator bias plays a role even though we used lags as instruments. Note that both MaCurdy's (column (4)) and Altonji's instruments (column (1)) circumvent the denominator bias. An explanation would be that measurement error is correlated across periods. In the next column, we use lagged levels and lagged differences of the log of annual labor income to circumvent this problem. The resulting coefficient is statistically significant, positive, and large, the Frisch elasticity is 0.483. The implied coefficient of relative risk aversion with respect to leisure is according to this estimate about

2.1. The fourth column shows the results using human capital related (HC) instruments following MaCurdy (1981) that include age, age squared, education, education squared, interactions between age and education, and age and education squared. The result is similar to the third column but less precisely estimated. Note that the number of observation is larger in the last columns, because only contemporary instruments are used.

In Table 3, we apply the same specifications to the SOEP data, except for the specification in Table 2, column (1), since the directly asked wage measure is only available in the PSID. Similar to the results for the US (see column (2) in Table 2), the coefficient of wage growth is significantly negative using the instruments based on the constructed wage measure. The magnitude is similar.

Using instruments based on lagged labor income and human capital, we obtain positive coefficients as with the PSID. However, their magnitudes are smaller. Using the SOEP all sets of instruments result in statistically significant coefficients due to the larger sample size. Using net wages, the results are very similar. In our preferred specification, columns (2) and (5), the implied $\rho$ is between 13 and 14.

Table 3: Replication and Alternative Regression of Growth Rate of *Hours* Using SOEP Data.

| | Gross | | | Net | | |
|---|---|---|---|---|---|---|
| | IV $\frac{inc_{t-1}}{h_{t-1}}$ (1) | IV $inc_{t-1}$ (2) | IV HC (3) | IV $\frac{inc_{t-1}}{h_{t-1}}$ (4) | IV $inc_{t-1}$ (5) | IV HC (6) |
| $\Delta\ln(inc/h)$ | -0.254*** | 0.075*** | 0.197*** | -0.193*** | 0.072*** | 0.154*** |
| | (0.006) | (0.009) | (0.029) | (0.005) | (0.012) | (0.027) |
| Constant | 0.004*** | -0.000 | -0.004*** | 0.003*** | -0.001 | -0.004*** |
| | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) | (0.001) |
| *N* | 45,480 | 45,480 | 57,035 | 41,225 | 41,005 | 53,115 |

*Notes:* Standard errors in parentheses. Dependent variable is the growth rate of hours. In column (1), instruments are the lagged level and the lagged difference of the log of the constructed hourly wage. In column (2), instruments are lagged levels and lagged differences of the log of annual labor income. In column (3), instruments are age, age squared, education, education squared, interactions between age and education, and age and education squared. In columns (4) to (6), marginal net wage is used instead of gross wage.

*Source:* Authors' calculations based on the SOEP (1996-2011).

In contrast to the specifications shown above, specification (9) allows to test implications of CRRA utility. The independent variables are $\Delta\ln(inc/h)$, $\frac{1}{2}(\Delta\ln W_t)^2$, and $\frac{1}{2L_{it-1}^2}(\Delta L_{it})^2$. If CRRA holds, the coefficient of the growth rate of hourly wage is $-1/\rho$, the coefficient of the second term is $1/\rho$, the inverse of the measure of relative risk aversion, and the coefficient of the square of the change of leisure is $1+\rho$, the parameter of relative prudence. If CRRA holds, the specification allows to quantify how risk averse and how prudent workers are. If the coefficients do not meet the implied restrictions of the utility function, CRRA is rejected.

The four columns of Table 4 correspond to those of Table 2, but the sets of independent variables and instruments are richer. Recall that the dependent variable is the growth rate of leisure and therefore the coefficient of wage growth should have the opposite sign. Only the coefficient of wage growth in column (4) is not similar across these tables. In the other three columns the coefficients have the opposite signs (as expected), but in all four columns the coefficients are statistically insignificant. In our preferred specification in column (3), the point estimate implies that $\rho = 15.4$ and the Frisch elasticity equals 0.065. This estimate lies between the ones obtained by Blundell and Walker (1986) and Browning et al. (1985).

The estimated coefficient for the measure of wage growth variability is in contrast to theoretical considerations negative throughout and statistically significant in the first three columns. The estimate of -0.77 in column (3) would imply a $\rho$ of -1.3. The coefficient of the measure of variability in leisure is across all specifications statistically significant and in line with the literature positive. The estimate of 1.86 in column (3) would imply a $\rho$ of 0.86. The corresponding Frisch elasticity would equal 1.2 which is larger than in most studies but in the range of the findings in Imai and Keane (2004). This study explicitly accounts for human capital which is an alternative explanation to precautionary labor supply for downward bias in other studies. The negative estimate for the constant implies impatience which corroborates a natural assumption often made (Deaton (1991)).

However, for CRRA utility to hold, the coefficient of $\frac{1}{2}(\Delta \ln W_t)^2$ must be the same as the one of wage growth but with the opposite sign. A further testable restriction is that this coefficient must equal the inverse of one minus the coefficient of leisure variability. Therefore, we test the non-linear restriction $H_{\text{CRRA}} : -1/\beta_1 = 1/\beta_2 = \beta_3 - 1$. The respective $\chi^2$ statistics and the corresponding p-values are given at the bottom of Table 4. For the first three specifications, we reject CRRA resoundingly. With the human capital related instruments, CRRA cannot be rejected, since the estimates are relatively imprecise.

This suggests that the widely used assumption of CRRA utility is a poor approximation of household behavior. Therefore alternative avenues should be explored. For instance, repeating a test like in our analysis based on more general preference structures like Epstein and Zin (1989, 1991) would be a fruitful endeavor. Moreover, alternatives to expected utility theory could result in a better explanation of intertemporal behavior. A straightforward way to extend the model without abandoning CRRA or expected utility theory is to specify a wage process that allows for different effects of deterministic and stochastic wage growth. Some work in this direction is shown in the appendix.

Table 4: Regression of Growth Rate of *Leisure* Using PSID Data.

| | IV $w_{t-1}$ (1) | IV $\frac{inc_{t-1}}{h_{t-1}}$ (2) | IV $inc_{t-1}$ (3) | IV HC (4) |
|---|---|---|---|---|
| $\Delta\ln(inc/h)$ | -0.051 | 0.052 | -0.065 | 0.428 |
| | (0.145) | (0.057) | (0.083) | (0.443) |
| $\frac{1}{2}(\Delta\ln W_t)^2$ | -1.129** | -0.485* | -0.770** | -1.591 |
| | (0.422) | (0.205) | (0.268) | (0.922) |
| $\frac{1}{2L_{it-1}^2}(\Delta L_{it})^2$ | 1.929*** | 1.594*** | 1.860*** | 1.193* |
| | (0.303) | (0.204) | (0.243) | (0.600) |
| Constant | -0.011 | -0.024*** | -0.019* | 0.006 |
| | (0.012) | (0.007) | (0.009) | (0.026) |
| $N$ | 1,277 | 1,277 | 1,277 | 1,277 |
| $H_{\text{CRRA}}: -1/\beta_1 = 1/\beta_2 = \beta_3 - 1$ | | | | |
| $\chi^2(2)$ | 44.67 | 11.05 | 30.35 | 0.88 |
| p-value | 0.000 | 0.004 | 0.000 | 0.6447 |

*Notes:* Standard errors in parentheses. Dependent variable is the growth rate of leisure (5,000 - hours). All independent variables are instrumented. In column (1), instruments are the lagged level and the lagged difference of the log of the directly asked hourly wage, squares, and lags of the square of the log and the growth rate of leisure. In column (2), instruments are identical except that they are based on the constructed hourly wage. In column (3), instruments are identical except that they are based on labor income. In column (4), instruments are lags of the square of the log and the growth rate of leisure, age, age squared, education, education squared, interactions between age and education, and age and education squared.

*Source:* Authors' calculations based on the PSID (1983-1985, 1988-1990, 1993-1995).

Table 5 reports estimates for Germany using the same specifications as in the last three columns of the previous table. Again, we add to the results based on gross wages those based on net wages. In contrast to the results for the US, the coefficient of wage growth is statistically significant and positive in all six columns. This is in contrast with the theoretical prediction. The coefficients for the squared wage terms are all statistically significant and similar to the ones obtained from the PSID data. As before, the signs of the coefficients are not in line with the theoretical considerations.

The leisure variability term implies a magnitude of $\rho$ between 0.16 and 1.06. Interestingly, the coefficients of our preferred specification in column (2) is very similar to the one for the US (columns (3) in Table 4).

For Germany, we have to reject the hypothesis that the restrictions implied by CRRA utility hold for all specifications.

Table 5: Regression of Growth Rate of *Leisure* Using SOEP Data.

| | Gross | | | Net | | |
|---|---|---|---|---|---|---|
| | IV $\frac{inc_{t-1}}{h_{t-1}}$ (1) | IV $inc_{t-1}$ (2) | IV HC (3) | IV $\frac{inc_{t-1}}{h_{t-1}}$ (4) | IV $inc_{t-1}$ (5) | IV HC (6) |
| $\Delta \ln(inc/h)$ | 0.235*** | 0.206*** | 0.636*** | 0.244*** | 0.296*** | 0.752*** |
| | (0.009) | (0.010) | (0.040) | (0.009) | (0.013) | (0.038) |
| $\frac{1}{2}(\Delta \ln W_t)^2$ | -1.100*** | -1.068*** | -3.616*** | -0.811*** | -1.005*** | -1.607*** |
| | (0.044) | (0.049) | (0.240) | (0.037) | (0.052) | (0.171) |
| $\frac{1}{2L_{it-1}^2}(\Delta L_{it})^2$ | 2.060*** | 2.064*** | 1.856*** | 1.953*** | 1.848*** | 1.156*** |
| | (0.037) | (0.038) | (0.115) | (0.036) | (0.041) | (0.093) |
| Constant | 0.005*** | 0.005*** | 0.065*** | 0.007*** | 0.014*** | 0.035*** |
| | (0.001) | (0.001) | (0.006) | (0.001) | (0.002) | (0.006) |
| $N$ | 45,480 | 45,480 | 46,390 | 41,225 | 41,005 | 41,840 |
| $H_{\text{CRRA}}: -1/\beta_1 = 1/\beta_2 = \beta_3 - 1$ | | | | | | |
| $\chi^2(2)$ | 2090.54 | 1461.60 | 154.60 | 1241.52 | 843.89 | 104.17 |
| p-value | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

*Notes:* Standard errors in parentheses. Dependent variable is the growth rate of leisure (5,000 - hours). All independent variables are instrumented. In column (1), instruments are the lagged level and the lagged difference of the log of the constructed hourly wage, squares, and lags of the square of the log and the growth rate of leisure. In column (2), instruments are identical except that they are based on labor income. In column (3), instruments are lags of the square of the log and the growth rate of leisure, age, age squared, education, education squared, interactions between age and education, and age and education squared. In columns (4) to (6), marginal net wage is used instead of gross wage.

*Source:* Authors' calculations based on the SOEP (1996-2011).

# 5 Summary and Conclusions

Deriving estimation equations from a theoretical model, we have studied the impact of wage uncertainty and variability in leisure on labor supply. This specification extends the conventional specifications and has the advantage that it allows to test implications of a widely used functional form (CRRA) for utility.

In a first step, we reproduce the results in Domeij and Flodén (2006) and show that the estimated Frisch elasticities are very sensitive to the choice of the instruments using PSID data. Then, we repeat the analysis with German data from the SOEP, to study differences in labor supply behavior across these countries. With our preferred specification we obtain statistically significant positive Frisch elasticities that are comparable to those obtained in the previous literature.

Using the extended estimation specification, we reject the implied restrictions of CRRA utility both for the US and for Germany. This suggests that widely used assumption of CRRA utility is a poor approximation of household behavior. Therefore alternative avenues should be explored. For instance, repeating a test like in our analysis based on more general preferences structures like Epstein and Zin (1989, 1991) would be a fruitful endeavor. Moreover, alternatives to expected utility theory could result in a better explanation of intertemporal behavior. A straightforward way to extend the model without abandoning CRRA or expected utility theory is to specify a wage process that allows for different effects of deterministic and stochastic wage growth.

# References


ALTONJI, J. G. (1986): "Intertemporal Substitution in Labor Supply: Evidence from Micro Data," *Journal of Political Economy*, 94(3), S176–S215. Cited on pages 2, 4, 5, 7, and 12.

ATTANASIO, O. P., AND H. LOW (2004): "Estimating Euler Equations," *Review of Economic Dynamics*, 7(2), 405–435. Cited on page 3.

BÉNABOU, R. (2002): "Tax and Education Policy in a Heterogeneous-Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?," *Econometrica*, 70(2), 481–517. Cited on page 8.

BLUNDELL, R., AND T. MACURDY (1999): "Labor supply: A review of alternative approaches," in *Handbook of Labor Economics*, ed. by O. Ashenfelter, and D. Card, vol. 3 of *Handbook of Labor Economics*, chap. 27, 1559–1695. Elsevier. Cited on page 2.

BLUNDELL, R., L. PISTAFERRI, AND I. SAPORTA-EKSTEN (2012): "Consumption Inequality and Family Labor Supply," Working Paper 18445, National Bureau of Economic Research. Cited on pages 5 and 8.

BLUNDELL, R., AND I. WALKER (1986): "A Life-Cycle Consistent Empirical Model of Family Labour Supply Using Cross-Section Data," *The Review of Economic Studies*, 53(4), 539–558. Cited on page 14.

BROWNING, M., A. DEATON, AND M. IRISH (1985): "A Profitable Approach to Labor Supply and Commodity Demands over the Life-Cycle," *Econometrica*, 53(3), pp. 503–544. Cited on page 14.

BROWNING, M., AND A. LUSARDI (1996): "Household Saving: Micro Theories and Micro Facts," *Journal of Economic Literature*, 34(4), pp. 1797–1855. Cited on pages 3 and 5.

CARROLL, C. D. (2001): "Death to the Log-Linearized Consumption Euler Equation! (And Very Poor Health to the Second-Order Approximation)," *The B.E. Journal of Macroeconomics*, 1(1), 1–38. Cited on pages 3 and 6.

CARROLL, C. D., AND A. A. SAMWICK (1997): "The nature of precautionary wealth," *Journal of Monetary Economics*, 40(1), 41 – 71. Cited on page 3.

——— (1998): "How Important Is Precautionary Saving?," *The Review of Economics and Statistics*, 80(3), 410–419. Cited on page 3.

DEATON, A. (1991): "Saving and Liquidity Constraints," *Econometrica*, 59(5), pp. 1221–1248. Cited on page 14.

DOMEIJ, D., AND M. FLODÉN (2006): "The Labor-Supply Elasticity and Borrowing Constraints: Why Estimates are Biased," *Review of Economic Dynamics*, 9(2), 242–262. Cited on pages 2, 3, 5, 8, 12, and 17.

DYNAN, K. E. (1993): "How Prudent are Consumers?," *Journal of Political Economy*, 101(6), pp. 1104–1113. Cited on page 3.

EPSTEIN, L. G., AND S. E. ZIN (1989): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57(4), 937–969. Cited on pages 14 and 17.

——— (1991): "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis," *Journal of Political Economy*, 99(2), 263–286. Cited on pages 14 and 17.

FELDSTEIN, M. S. (1969): "The Effects of Taxation on Risk Taking," *Journal of Political Economy*, 77(5), 755–764. Cited on page 8.

FLODÉN, M. (2006): "Labour Supply and Saving Under Uncertainty*," *The Economic Journal*, 116(513), 721–737. Cited on pages 2, 4, and 22.

FOSSEN, F. M., AND D. ROSTAM-AFSCHAR (2013): "Precautionary and Entrepreneurial Savings: New Evidence from German Households," *Oxford Bulletin of Economics and Statistics*, 75(4), 528–555. Cited on page 3.

GRUBER, J. (2013): "A Tax-Based Estimate of the Elasticity of Intertemporal Substitution," 03(01). Cited on pages 3 and 5.

GUISO, L., T. JAPPELLI, AND D. TERLIZZESE (1992): "Earnings uncertainty and precautionary saving," *Journal of Monetary Economics*, 30(2), 307 – 337. Cited on page 3.

HEATHCOTE, J., K. STORESLETTEN, AND G. L. VIOLANTE (2014): "Optimal Tax Progressivity: An Analytical Framework," NBER Working Paper 19899, National Bureau of Economic Research. Cited on page 8.

IMAI, S., AND M. P. KEANE (2004): "Intertemporal Labor Supply and Human Capital Accumulation," *International Economic Review*, 45(2), 601–641. Cited on page 14.

JESSEN, R., D. ROSTAM-AFSCHAR, AND V. STEINER (2015): "Getting the Poor to Work: Three Welfare Increasing Reforms for a Busy Germany," mimeo, Freie Universitaet Berlin. Cited on page 8.

KEANE, M. P. (2011): "Labor supply and taxes: A survey," *Journal of Economic Literature*, 49(4), 961–1075. Cited on page 2.

KIMBALL, M. S. (1990): "Precautionary Saving in the Small and in the Large," *Econometrica*, 58(1), pp. 53–73. Cited on pages 2, 4, and 22.

LOW, H. (2005): "Self-Insurance in a Life-Cycle Model of Labor Supply and Savings," *Review of Economic Dynamics*, 8(4), 945–975. Cited on pages 2, 3, and 6.

LUDVIGSON, S., AND C. H. PAXSON (2001): "Approximation Bias in Linearized Euler Equations," *The Review of Economics and Statistics*, 83(2), pp. 242–256. Cited on page 6.

LUSARDI, A. (1998): "On the Importance of the Precautionary Saving Motive," *The American Economic Review*, 88(2), pp. 449–453. Cited on page 3.

MACURDY, T. E. (1981): "An Empirical Model of Labor Supply in a Life-Cycle Setting," *Journal of Political Economy*, 89(6), 1059–85. Cited on pages 2, 4, 7, 12, and 13.

MULLIGAN, C. B. (1998): "Pecuniary Incentives to Work in the United States during World War II," *Journal of Political Economy*, 106(5), pp. 1033–1077. Cited on page 2.

PARKER, S. C., Y. BELGHITAR, AND T. BARMBY (2005): "Wage Uncertainty and the Labour Supply of Self-Employed Workers*," *The Economic Journal*, 115(502), C190–C207. Cited on page 2.

PETERMAN, W. B. (2014): "Reconciling Micro and Macro Estimates of the Frisch Labor Supply Elasticity: A Sensitivity Analysis," Feds working paper, Federal Reserve Board of Governors. Cited on page 4.

PISTAFERRI, L. (2003): "Anticipated and Unanticipated Wage Changes, Wage Risk, and Intertemporal Labor Supply," *Journal of Labor Economics*, 21(3), pp. 729–754. Cited on pages 2 and 5.

PRATT, J. W. (1964): "Risk Aversion in the Small and in the Large," *Econometrica*, 32(1/2), pp. 122–136. Cited on pages 4 and 22.

WAGNER, G. G., J. R. FRICK, AND J. SCHUPP (2007): "The German Socio-Economic Panel Study (SOEP): Scope, Evolution and Enhancements," *Schmollers Jahrbuch*, 127(1), 139–169. Cited on page 8.

ZILIAK, J. P., AND T. J. KNIESNER (1999): "Estimating Life Cycle Labor Supply Tax Effects," *Journal of Political Economy*, 107(2), pp. 326–359. Cited on page 8.

# A Appendix

The real wage is assumed to have both transitory and permanent lognormally distributed shocks, where individual subscripts are suppressed:

$$\ln w_t = \ln w_0 + \alpha_1 t + \alpha_2 t^2 + \ln^P w_t + \nu_t, \nu_t \sim N\left(\frac{-\sigma_{\nu t}^2}{2}, \sigma_{\nu t}^2\right), \tag{10}$$

$$\ln^P w_t = \ln^P w_{t-1} + p_t, p_t \sim N\left(\frac{-\sigma_{pt}^2}{2}, \sigma_{pt}^2\right).$$

The log wage at age $t$ follows a deterministic trend plus the log of the permanent wage plus a transitory shock $\nu_t$. The transitory shock is assumed to be i.i.d. We allow for the variance of transitory and permanent shocks to depend on $t$ and to differ between individuals.

In first differences the wage processes of period $t+1$ is

$$\Delta \ln w_{t+1} = \ln w_0 + \alpha_1(t+1) + \alpha_2(t+1)^2 + \ln^P w_t + p_{t+1} + \nu_{t+1} \tag{11}$$

$$\ln w_0 - \alpha_1 t - \alpha_2 t^2 - \ln^P w_t - \nu_t$$

$$\alpha_1 + (2t+1)\alpha_2 + p_{t+1} + \nu_{t+1} - \nu_t.$$

Collecting terms gives

$$\Delta \ln w_{t+1} = \alpha_1 + (2t+1)\alpha_2 + p_{t+1} + \nu_{t+1} - \nu_t. \tag{12}$$

Taking expectations yields

$$E_t[\Delta \ln w_{t+1}] = \alpha_1 + (2t+1)\alpha_2. \tag{13}$$

Squaring (12) results in

$$(\Delta \ln w_{t+1})^2 = \alpha_1 + (2t+1)\alpha_2 + p_{t+1} + \nu_{t+1} - \nu_t \tag{14}$$

and taking expectations gives

$$E_t[(\Delta \ln w_{t+1})^2] = \alpha_1^2 + 2(2t+1)\alpha_1\alpha_2 + [(2t+1)\alpha_2]^2 + \sigma_p^2 + 2\sigma_\nu^2. \tag{15}$$

We substitute for $E_{t_0}\Delta \ln W_{t+1}$ and $E_{t_0}[(\Delta L_{t+1})^2]$ from the wage process (10) and simplify:

$$E_{t_0}\Delta L_{t+1} \approx \upsilon(r-\delta) - \upsilon[\alpha_1 + \alpha_2 - \frac{1}{2}(\alpha_1+\alpha_2)^2] \tag{16}$$

$$+\varkappa\frac{1}{2}E_{t_0}[(\Delta L_{t+1})^2]+\upsilon 2\alpha_1\alpha_2 t+2\upsilon\alpha_2^2t^2+\upsilon\sigma_{pt}^2+\upsilon\sigma_{vt}^2,$$

where $\upsilon=-\frac{u_L}{u_{LL}}$, the inverse of the measure of absolute risk aversion (Pratt 1964), and $\varkappa=-\frac{u_{LLL}}{u_{LL}}$, the measure of absolute prudence (Kimball (1990); Flodén (2006)).

Assuming that expectations coincide with realizations plus a mean zero error term $\varepsilon$ and collecting terms, one can rewrite

$$\Delta L_{t+1}=\beta_0+\beta_1\frac{1}{2}\sigma_{\Delta L}^2+\beta_2 t+\beta_3 t^2+\beta_4\frac{1}{2}\sigma_{\Delta W}^2+\varepsilon_t, \tag{17}$$

where $\beta_0=\upsilon(r-\delta)-\upsilon[\alpha_1+\alpha_2-\frac{1}{2}(\alpha_1+\alpha_2)^2]$, $\beta_1=\varkappa>0$, $\sigma_{\Delta L}^2=E_{t_0}[(\Delta L_{t+1})^2]$, $\beta_2=\upsilon 2\alpha_1\alpha_2<0$, $\beta_3=2\upsilon\alpha_2^2>0$, $\beta_4=\upsilon>0$, $\sigma_{\Delta W}^2=\sigma_p^2+2\sigma_v^2$.

Expanding (17) yields the estimable equation

$$E_{t_0}\Delta L_{t+1}\approx\beta_0+\beta_1\frac{1}{2L}\sigma_{\Delta L}^2+\beta_2 tL+\beta_3 t^2L+\beta_4\frac{1}{2}\sigma_{\Delta W}^2L+\beta_5 L+\varepsilon, \tag{18}$$

where $\beta_1=L\varkappa>0$, $\sigma_{\Delta L}^2=E_{t_0}[(\Delta L_{t+1})^2]$, $\beta_2=\frac{\upsilon}{L}2\alpha_1\alpha_2<0$, $\beta_3=\frac{\upsilon}{L}2\alpha_2^2>0$, $\beta_4=\frac{\upsilon}{L}>0$, $\sigma_{\Delta W}^2=\sigma_p^2+2\sigma_v^2$, $\beta_5=\frac{\upsilon}{L}(r-\delta)-\frac{\upsilon}{L}[\alpha_1+\alpha_2-\frac{1}{2}(\alpha_1+\alpha_2)^2]$.